Abstract—In latest CMOS technologies, Random Telegraph Noise (RTN) has emerged as an important challenge for SRAM design. Due to rapidly shrinking device sizes and heightened variability, analytical approaches are no longer applicable for characterising the circuit-level impact of non-stationary RTN. Accordingly, this paper presents SAMURAI, a computational method for accurate, trap-level, non-stationary analysis of RTN in SRAMs. The core of SAMURAI is a technique called Markov Uniformisation, which extends stochastic simulation ideas from the biological community and applies them to generate realistic traces of non-stationary RTN in SRAM cells. To the best of our knowledge, SAMURAI is the first computational approach that employs detailed trap-level stochastic RTN generation models to obtain accurate traces of non-stationary RTN at the circuit level. We have also developed a methodology that integrates SAMURAI and SPICE to achieve a simulation-driven approach to RTN characterisation in SRAM cells under (a) arbitrary trap populations, and (b) arbitrarily time-varying bias conditions. Our implementation of this methodology demonstrates that SAMURAI is capable of accurately predicting non-stationary RTN effects such as write errors in SRAM cells.

I. INTRODUCTION AND MOTIVATION

SRAMs\(^1\) find application in several domains including CPU caches, LCD screens and on-chip memories for both ASICs and FPGAs. This is because SRAMs offer several advantages over their dynamic counterparts: they are much faster, they consume less power and they do not need complex circuitry to periodically refresh their states.

Fig. 1 (left) shows the schematic for an SRAM cell that stores one bit (\(Q\)). The core of this SRAM cell is a cross-coupled pair of inverters formed by the four transistors M3-M6 (Fig. 1 right). The external circuit can read and write \(Q\) by controlling the pass transistors M1 and M2.

A. RTN as an emerging SRAM design challenge

In deeply scaled technologies, several non-idealities limit the minimum supply voltage (\(V_{dd}\)) under which an SRAM cell can be operated. Fig. 2 depicts the increase in \(V_{dd}\) necessitated by different non-idealities, as a function of CMOS technology [1]. Each CMOS technology is represented by a stacked bar, to which the effects of different non-idealities such as (a) \(V_T\) shifts due to global and local parameter variations, (b) Negative Bias Temperature Instability (NBTI), and (c) Random Telegraph Noise (RTN) are successively added (on top of the nominal supply voltage that overcomes static noise). Also included is a plot (the downward sloping dashed line) depicting the scaling of \(V_{dd}\) at different technologies.

As seen from the figure, the impact (in \(V_{dd}\) terms) of RTN on SRAMs has been steadily increasing under continued CMOS scaling. Moreover, coming on top of the other non-idealities, it is the incremental contribution made by RTN that is poised to push the minimum supply voltage over the dashed line that represents \(V_{dd}\) scaling. If this happens, RTN would reduce the SRAM design margin to zero and also render any further voltage scaling impossible. Therefore, to ensure continued CMOS scaling, RTN is of enormous significance even though its magnitude is small compared to other non-idealities.

B. Towards overcoming RTN

Although RTN is a severe limiting factor (as seen from Fig. 2), the following three observations suggest strategies for coping with it:

Correlation between RTN and NBTI: Recent evidence [1] suggests that RTN and NBTI are positively correlated. As a result, the total design margin impact of RTN and NBTI, taken together, is likely to be smaller than the sum of their individual design margin impacts [1]. In Fig. 2, this corresponds to an overlap between the top two boxes of each stack, which, if accurately taken into account, would enable more design choices.

Pessimism of stationary RTN analysis (hence the need for non-stationary analysis): From measurement data, it is well-known that stationary RTN analysis harbours considerable pessimism (the difference between predicted and observed noise power is sometimes as high as 15dB) [2], [3]. Hence it is likely that non-stationary RTN analysis techniques (such as the one we propose in this paper) would open up more design choices, by virtue of being more accurate in real-world (non-stationary) operating environments.

The case for computational RTN characterisation: RTN is caused by the random capture and release of charge carriers by traps located in a MOS transistor’s oxide layer [4], [5]. Analytical approaches to RTN characterisation are based on the assumption that a large number of active traps exist in the dielectric. Under this assumption, statistical averaging

---

\(^1\)Static Random Access Memories
shows that RTN obeys 1/f characteristics [4], [5]. However, with increasingly small device sizes, this assumption is no longer valid [6]–[8]. Indeed, detailed models for trap profiling (corroborated by measured data) suggest that in deeply scaled CMOS technologies, only about 5-10 traps are active at any given bias point [6], [7]. As a result, the analytical 1/f fit is often a poor model for highly scaled devices.

Fig. 3 illustrates this point using spectral density plots for 25 device instances (randomly sampled using the trap profiling model of [6], and held at constant bias) from two CMOS technologies. While the analytical solution is clearly a good fit for the older technology (left), it completely fails to capture the trap profiles of devices in the newer technology (right).

Although Fig. 3 shows that analytical approaches are no longer applicable, the encouraging observation is that this represents a clear case for computational approaches based on stochastic simulation [9] of trap activities. These approaches (such as the one proposed in this paper) are ideally suited to handle the small trap populations in today’s highly scaled devices. Indeed, computational approaches actually take advantage of the small trap populations to achieve greater efficiency.

C. What does accurate RTN characterisation entail?

The design of RTN-tolerant future generation SRAMs depends critically on our ability to leverage the above three observations. For this we need: An accurate model for RTN: It is known that both RTN and NBTI originate due to traps in the MOS oxide layer. The correlation between RTN and NBTI is most likely due to this common root cause [1]. Therefore, an RTN model based on first principles (i.e., the capture and emission of electrons by traps in the oxide layer) is likely to succeed in accurately capturing the NBTI correlation. Whereas detailed equations describing RTN generation from first principles at the device level are already available [5], [6], this paper, to the best of our knowledge, is the first attempt at incorporating such sophisticated models into a tool for RTN characterisation at the circuit level.

A computational method for predicting the circuit-level impact of non-stationary RTN: Today, the most advanced computational approach for RTN is that of Ye et. al. [10], which works by generating RTN-like waveforms starting from ideal white-noise sources. The main advantage of this method is that it integrates RTN simulation with SPICE-level circuit simulation. However, a key drawback of this method is that it is incapable of taking into account the bias-dependent, non-stationary statistics of RTN, which play a crucial role during SRAM operation (see next subsection). Moreover, the white noise sources drastically reduce the efficiency of this method from a simulation perspective. Indeed, to date, the only reported SRAM application of this method has been to analyse the simplest case of a single trap in an entire SRAM cell, and that too operating under constant bias assumptions.

By contrast, we develop a technique (§III) for generating genuinely non-stationary RTN. This technique, based on uniformisation [11]–[13] of a trap-level Markov chain model (§II), provably generates RTN traces that are (stochastically) exactly identical to the RTN physically measured on fabricated SRAMs. Hence the title of this paper, SAMURAI, which stands for SRAM Analysis by Markov Unification with RTN Awareness Incorporated. While being a computational method based on trap-level first principles, SAMURAI is capable of accurately simulating non-stationary RTN at the circuit level under (a) arbitrary trap populations, and (b) arbitrarily time-varying bias conditions. Moreover, we are able to integrate SAMURAI with SPICE, without encountering efficiency issues, to conduct full-fledged RTN analysis of SRAMs with varying trap populations under realistic, non-stationary operating environments.

D. Non-stationarity of RTN in SRAMs: A closer look

We have already mentioned that non-stationary analysis, being less pessimistic, often enables more design choices. We now illustrate the critical importance of non-stationary RTN, in the context of SRAMs.

In an SRAM cell, RTN can produce two adverse effects [14], [15]: (1) it can slow down write operations, and (2) it can cause write errors.

Consider the pass transistor M1, whose RTN can be modelled as a current source $I_{RTN}$ that opposes the nominal transistor current, as illustrated in Fig. 4 (right). The top three waveforms in Fig. 4 (left) are the signals applied to write a 1 to the SRAM cell, while the bottom two waveforms (numbered 1 and 2) represent two possible $I_{RTN}$ ”glitches”.

![Fig. 4. RTN in pass transistor M1 modelled as a glitch $I_{RTN}$ while a 1 is being written to the SRAM cell.](image)

Fig. 5 shows BSIM-4 SPICE simulations for the different $I_{RTN}$ scenarios. The top portion shows that in the absence of $I_{RTN}$ glitches, the signals

2RTN-induced SRAM read failures have also been reported [16]. SAMURAI is capable of predicting these too; however, due to space constraints, we do not discuss read failures in this paper.
At any given moment, an oxide trap can be in one of two possible states (illustrated in Fig. 8 (left)) that integrates SAMURAI and SPICE to enable accurate RTN analysis of SRAMs. We have implemented each step of this methodology and obtained results showing that SAMURAI does indeed fill traps. Specifically, we have developed a simulation-driven methodology and simulation of non-stationary computational RTN characterisation technique, with application to SRAM design. Against this background, in this paper, we develop SAMURAI, a computational tool for accurate modelling and simulation of non-stationary RTN. Starting from first principles (the capture/release of electrons by MOS oxide traps), SAMURAI generates accurate RTN traces for entire circuits (such as SRAMs) under arbitrary trap populations and arbitrarily time-varying bias conditions.

SAMURAI is a computational approach based on Markov Uniformisation [11]–[13], which is an extension of Gillespie’s stochastic simulation algorithm [9] to handle non-stationarity. Such computational approaches are well-known in the biological community, where they are used for accurate simulation of biochemical reactions involving a small number of molecules [9], [17], [18]. Analogously, we use SAMURAI for accurate simulation of RTN in SRAMs involving a small number of traps. Specifically, we have developed a simulation-driven methodology (illustrated in Fig. 8 (left)) that integrates SAMURAI and SPICE to enable accurate RTN analysis of SRAMs. We have implemented each step of this methodology and obtained results showing that SAMURAI does indeed accurately predict non-stationary RTN effects in SRAMs. We have also validated SAMURAI against analytical results known for stationary RTN.

II. THE RTN GENERATION MECHANISM: FROM TRAPS TO MARKOV CHAINS

A. Origins of RTN: Traps in the dielectric

As mentioned before, RTN is caused by the random capture and release of electrons by traps located in the MOS oxide layer (as shown in Fig. 6). At any given moment, an oxide trap can be in one of two possible states, (a) filled (i.e., the trap has captured an electron from the inversion layer), or (b) empty (i.e., the trap has released any previously captured electron back into the inversion layer). For a given trap, the evolution of its state over time (between filled and empty) is inherently random, i.e., it is a stochastic process. The parameters of this stochastic process depend on three factors: (a) the vertical distance $y_{tr}$ of the trap from the oxide-semiconductor interface, (b) the trap energy level $E_{tr}$, and (c) the instantaneous gate bias $V_{gs}(t)$ of the device [6]. It is this dependence on $V_{gs}(t)$ that makes the trap statistics, and hence the induced RTN, non-stationary.

Fig. 6. Left: Traps in a MOS transistor’s oxide layer. Right: A time-inhomogeneous two-state Markov chain model for a single trap.

B. Traps as time-inhomogeneous two-state Markov chains

Given that a trap is empty (filled) at time $t$, the probability that it will become filled (empty) by time $t + dt$ (i.e., it will capture (emit) an electron in the small time interval $dt$) is given by $\lambda_e(t) dt$ ($\lambda_f(t) dt$), where $\lambda_e(t)$ ($\lambda_f(t)$) is a time-varying function called the capture (emission) propensity function of the trap.

The above stochastic model governing the activity of a single trap can be described by a two-state time-inhomogeneous Markov chain, as shown in Fig. 6 (right). The two states in this Markov chain are designated 0 (empty) and 1 (filled), while transitions between states are labelled by the corresponding propensity functions. The functions $\lambda_e(t)$ and $\lambda_f(t)$ depend on the instantaneous bias $V_{gs}(t)$, and also on the trap characteristics $y_{tr}$ and $E_{tr}$. Detailed equations describing these dependencies can be found in [6], from which we obtain:

$$\lambda_e(t) + \lambda_f(t) = \frac{1}{\beta^\gamma}$$

$$\beta(t) = \frac{\lambda_e(t)}{\lambda_f(t)} = g e^{\frac{y_{tr} - E_{tr}}{kT}}$$

From the above equations, it is seen that the sum $\lambda_e + \lambda_f$ is constant with time, depending only on $y_{tr}$, the time constant $\tau_0$ for traps at the silicon interface and the tunnelling coefficient $\gamma$. However, the ratio $\beta = \lambda_e/\lambda_f$ at time $t$ is a complex function (whose exact form is given in [6]) of the instantaneous bias $V_{gs}(t)$. Because the ratio $\beta$ is time-varying, the trap statistics are non-stationary. (Here $g$ is the trap degeneracy factor, $k$ is the Boltzmann constant and $T$ is the temperature.)

C. From trap occupancies to RTN currents

The evolution of the state of a single trap is governed by the stochastic model above. In a MOS transistor, there may exist multiple such traps. Given a trap occupancy function for the device, i.e., a description of how the state of each trap evolves over time, detailed models exist for predicting the noise current $I_{RTN}(t)$ in the device. For example, one model (currently used by SAMURAI) is the following equation [19]:

$$I_{RTN}(t) = \frac{L_d(t)}{W L N(t)} N_{filled}(t)$$

where $L_d$ is the nominal drain current (without RTN), $W$ and $L$ are the device dimensions, $N$ is the number density of charge carriers and $N_{filled}(t)$ is the number of device traps filled at time $t$ (which can be calculated from the trap occupancy function). More complex models have also been suggested (e.g., [20]) which, if needed, can be incorporated into SAMURAI just as easily.

III. THE SAMURAI CORE: RTN TRACE GENERATION BY MARKOV UNIFORMISATION

The previous section described how to compute the RTN current given the trap occupancy function. This section describes a technique for computing the trap occupancy function.

For a device with multiple traps, each trap can be thought of as a separate two-state time-inhomogeneous Markov chain. In order to generate real-
istic RTN traces under time-varying bias conditions, SAMURAI carries out non-stationary stochastic simulation (Algorithm 1) of each of these Markov chains. The trap occupancy function thus computed is used to generate a realistic RTN trace $I_{RTN}(t)$.

Algorithm 1: Non-stationary RTN generation in SAMURAI

```
Input: Trap profile, Bias \{V_{gr}(t), I_d(t) \ldots \}, t_0, t_f
Output: Realistic $I_{RTN}(t)$ trace in time interval \([t_0,t_f]\)

1. foreach trap tr in the device do
2.   compute $\lambda_c(t), \lambda_e(t), t \in \{t_0,t_f\}$, for tr (use Eq. (1), (2));
3.   $\lambda^* = \lambda_c(t_0) + \lambda_e(t_0);
4.   curr_time = t_0; curr_state = tr.init_state;
5.   times = [curr_time]; states = [curr_state];
6.   while curr_time < t_f do
7.     next_cand_time = curr_time + exprand(1/\lambda^*);
8.     if curr_time > t_f then break;
9.     if curr_state == l then
10.    $\lambda_{next} = \lambda_c(curr_time)$
11.   else $\lambda_{next} = \lambda_e(curr_time)$
12.   end
13.   bool change_the_state = rand() < $\lambda_{next}/\lambda^*$;
14.   if change_the_state then
15.     times.append(curr_time);
16.     states.append(curr_state);
17.     curr_state = (curr_state == l) ? 0 : 1;
18.     times.append(curr_time);
19.     states.append(curr_state);
20.   end
21. end
22. end
23. compute $I_{RTN}(t)$ from trap_occupancy[tr] (use Eq. (3))
```

Algorithm 1 takes as input, (a) the trap profile of the device (i.e., the position $y_r$, and energy $E_r$, of each trap), and (b) the time-varying bias conditions. It produces as output an $I_{RTN}(t)$ trace whose statistics are exactly identical to the time-varying RTN statistics under the non-stationary model of the previous section.

Briefly, the algorithm works by generating more trap activity than necessary, and then discarding some of the generated activity so that the time-varying trap statistics are exactly preserved. Line 3 computes $\lambda^*$, which is an upper bound on the functions $\lambda_c(t)$ and $\lambda_e(t)$. In each iteration of the while loop (line 6), a candidate event is generated (line 7) corresponding to a stationary two-state Markov chain with both propensities set to $\lambda^*$. Thus, the original non-stationary Markov chain is first uniformised into a stationary (but high rate) Markov chain. In line 15, a probabilistic decision is made to either keep or discard the generated event, which exactly restores the non-stationarity of the original Markov chain. That this algorithm exactly preserves the original Markov chain’s non-stationarity is proved in [11]-[13].

Although Algorithm 1 generates $I_{RTN}(t)$ traces only for a single device, it can be straightforwardly extended to investigate the effect of RTN on entire SRAM cells (as we do in the next section).

IV. RESULTS

In this section we present two kinds of results, (a) validation results that demonstrate excellent agreement between SAMURAI’s predictions and analytical expressions known for stationary RTN, and (b) simulation results showing that SAMURAI can accurately predict the effects of non-stationary RTN in SRAM cells.

A. Validation results

SAMURAI is primarily intended for non-stationary RTN analysis under arbitrarily time-varying bias conditions. Although analytical expressions are not available for such a general case, they are known for the restricted constant bias case [3], [5]. Here we validate SAMURAI against these expressions for a wide range of trap configurations.

- We run three validation experiments, using typical values for the 3 parameters that affect the trap capture/release statistics, namely, $V_{gr}$, $E_r$, and $y_r$. In each experiment, we fix two of these parameters and sweep the third over an appropriate range. Hence we generate a variety of trap configurations, which are then simulated under constant gate bias using Algorithm 1.

- Algorithm 1 returns a trace $I_{RTN}(t)$. From this trace, we numerically estimate the autocorrelation function $R(\tau) = E[I_{RTN}(t)I_{RTN}(t+\tau)]$.

- We also translate the above time-domain results into frequency domain by computing the stationary power spectral density $S(f)$ numerically from $R(\tau)$.

The results are presented in Fig. 7 (a to f). In all these plots, $\tau$ is measured in seconds, $R(\tau)$ in $A^2$, all frequencies are in $Hz$, and all spectral densities are in $A^2/Hz$. From Fig. 7, it is seen that the RTN traces predicted by SAMURAI closely match analytical expressions in both the time domain (autocorrelation plots (a)-(c)) and the frequency domain (spectral density plots (d)-(f)).

B. SRAM simulation results

Fig. 8 (left) shows a flowchart illustrating our methodology for analysing non-stationary RTN in SRAMs. This methodology combines SAMURAI with SPICE, resulting in an accurate, trap-level, simulation-driven strategy for SRAM design in the presence of RTN.

- First we simulate the SRAM cell (on a test pattern of reads and writes) without RTN in SPICE. This enables the generation of time-varying biases (to be used as input to SAMURAI).

- Next, we use SAMURAI to generate RTN traces for each transistor in the SRAM cell, under the biases obtained from SPICE. In addition to the biases, this step requires a trap profile for each device, which is either obtained from measurement data [7] or generated using statistical trap profiling models proposed in the literature [6].

- We model the RTN traces generated above as current sources between the drain and source of each device (similar to Fig. 4 (right)). We again carry out SPICE simulation of the SRAM cell (on the same test pattern), this time including the $I_{RTN}$ of each transistor.

- If the second SPICE simulation predicts write errors or unacceptable slowdown in SRAM operation, it is immediately clear that the SRAM cell is compromised due to RTN (either $V_{dd}$ must be increased or the SRAM cell must be re-designed). Otherwise, the analysis is repeated with a new test pattern (or a conclusion is reached that the SRAM cell is indeed robust to RTN).

In Fig. 8 (right), we have implemented each step of the above methodology (in 90nm technology using BSIM-4 device models and SpiceOPUS [21] for circuit simulation) and presented the results in 5 plots (labelled (a)-(e)) alongside the flowchart, with arrows pointing from the flowchart stages to the relevant plots. We have illustrated the entire methodology using the bit pattern [1,1,0,1,0,1,0,0,1] written to the SRAM cell. As seen from plot (a), this bit pattern is successfully written to the SRAM cell if
Fig. 7. Plots showing that the RTN traces generated by SAMURAI closely match analytical predictions in both the time domain (autocorrelation plots (a)-(c)) and the frequency domain (spectral density plots (d)-(f)).

there is no RTN.

Plots (b) and (c) show the trap occupancy functions returned by SAMURAI for transistors M5 and M6 respectively (here we have used the statistical trap profiling model proposed in [6]). As is to be expected [3], the transistor M5 (whose gate voltage is \( Q \)) exhibits a high degree of trap activity when \( Q \) is high, but very little trap activity when \( Q \) is low. The opposite is true for transistor M6, whose gate voltage is \( \overline{Q} \). Thus, SAMURAI is able to accurately predict the time-varying (non-stationary) statistics of trap activity under rapidly switching bias conditions. (For illustrative purposes, an exploded view of the trap activity during a small time interval from each of these plots is also shown.)

Plot (d) shows the RTN trace generated by SAMURAI for the transistor M2. Plot (e) shows a SPICE simulation of the SRAM cell (on the same bit pattern) after including the \( I_{RTN} \) current sources for each transistor. Here we wanted to demonstrate that SAMURAI is indeed capable of predicting non-stationary effects such as write errors in SRAM cells. However, such failures are extremely rare events. Therefore, for illustrative purposes, we have scaled the \( I_{RTN} \) trace of each transistor by a factor of 30, which immediately produced a write error (as seen from plot (e)). In more deeply scaled CMOS technologies (e.g., 22nm), such artificial RTN scaling would not be necessary. Moreover, if other sources of variability (e.g., local/global parameter variations, NBTI etc.) are taken into account, the incremental effect of RTN would be sufficient to produce a bit error (without artificial scaling). Also, instead of scaling \( I_{RTN} \), we note that a similar effect can be achieved by adopting accelerated RTN testing techniques for SRAMs (such as the one outlined in [14]). That is, SAMURAI should be run on the SPICE response predicted for the SRAM cell under the biases suggested by accelerated testing techniques.

V. CONCLUSIONS AND FUTURE WORK

In this paper, we have presented SAMURAI, a computational technique that enables accurate, trap-level, non-stationary analysis of RTN in SRAMs. The core of SAMURAI is a procedure called Markov Uniformisation, which extends stochastic simulation ideas from the biological community to the RTN characterisation problem in SRAMs. Starting from a two-state time-inhomogeneous Markov chain model for each device trap, we have demonstrated that SAMURAI is capable of generating non-stationary RTN traces for entire circuits under (a) arbitrary trap populations, and (b) arbitrarily time-varying bias conditions. To the best of our knowledge, SAMURAI is the first computational tool that incorporates sophisticated, trap-level, stochastic RTN models into a methodology for circuit level RTN characterisation. Where analytical expressions are available, SAMURAI closely matches their predictions. And where analytical expressions are not available, SAMURAI still enables accurate RTN characterisation by generating realistic RTN traces.

Against the above accomplishments, we now identify four directions for future research:

1. Bi-directionally coupled RTN simulation. Throughout this paper, we have assumed that the biases for RTN trace generation can be pre-computed by a SPICE simulation. However, in reality, RTN cannot be isolated from the rest of the circuit, i.e., both RTN and the circuit states evolve together, with RTN modulating the circuit voltages/currents and the circuit simultaneously modulating the stochastic processes governing RTN generation, thereby forming a bi-directionally coupled system. For the future, our aim is to accurately simulate such “higher order” effects associated with RTN.

2. Accounting for other sources of variability. In this paper, we have discussed the stand-alone impact of RTN on SRAMs. In reality, RTN occurs on top of other non-idealities such as static noise, local/global parameter variations, NBTI etc. In future, we would like to properly account for all these variabilities within our simulation tool.

3. Statistical analysis of RTN for entire SRAM arrays. In this paper, we
have considered the effect of RTN on a single SRAM cell. We view this as the first step towards predicting the bit-error impact of RTN on entire SRAM arrays, which are made up of thousands of SRAM cells that are subject to local and global parameter variations.

4. Applications beyond SRAMs. RTN has been shown to adversely affect many circuits other than SRAMs. For instance, RTN is thought to be responsible for Variable Retention Time (VRT) in DRAMs [22, 23]. RTN is also known to impact ring oscillators [3]. We also conjecture that RTN causes cycle slipping in Phase Locked Loops (PLLs). In future, we would like to extend SAMURAI to conduct RTN analysis for all these different circuits.

Acknowledgements: We would like to thank Yasumasa Tsukamoto of Renesas Electronics Corp. for Fig. 2 (and many useful discussions, besides). Support from the Semiconductor Research Corporation (SRC), in particular, for his encouragement of this work.

References