**LTI Systems**

**Impulse Response:** $h[n]$ characterises $u[n] \rightarrow y[n]$ fully via convolution:

$y[n] = u[n] \ast h[n]$.

If you zero-pad $u$ and $h$ (to $u_p$ and $h_p$), then

Circular convolution = convolution.

i.e., $u \ast h = u_p \circledast h_p$.

**DFT**

Complex conjugacy:

$x[n]$ real $\iff x[n] = x^*[n]$.

DFT transforms TD. to F-D.:

i.e.) sinusoid at freq. $\omega_k = X_k^*(scaled)$ phasor.

DFT coeffs. $X_k$, can be filtered easily:

$y[n] = X_k \cdot H_k$; $Y_k$ are filtered DFT coeffs.

Circular convolution in TD. = ELEM. BY ELEM. MULTIPLICATION IN F-D.:

i.e., $x[n] \circledast h[n] = F_f^{-1}\left(F_f(x[n]) \cdot F_f(h[n])\right)$.

Circular convolution can be done fast (via FFT).

DFT can be used for fast convolution via zero-paddng.

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**Example:** Convolution via padding and DFT

$N = 2$, $N = 2M + 1 = 5$.

$\tilde{u} = [1 \ 0 \ 1 \ 0 \ 1]^T$

$\tilde{h} = [5 \ 4 \ 3 \ 2 \ 1]^T$

$\tilde{y} = \tilde{u} \circledast \tilde{h} = [5 \ 4 \ 8 \ 6 \ 9]^T$

$\sum_{i=0}^{N-1} u[i]h[i]$

$\tilde{u}_p = [1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0]^T \rightarrow \tilde{u}_p = F_F \tilde{u}_p$,

$\downarrow \tilde{y}_p = \tilde{u}_p \ast \tilde{h}_p$

$\tilde{F}_F^{-1}$.

$\tilde{y} = \tilde{y}_p$.

$\tilde{F}_F Y = ? \tilde{y}?$
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